The Fundamentals of MTF, Wiener Spectra, and DQE

Robert M Nishikawa

Kurt Rossmann Laboratories for Radiologic Image Research
Department of Radiology, The University of Chicago

Motivation

Goal of radiology: to diagnosis and treat disease by

Role of Medical Physicist: to help maximize patient benefit while minimizing the cost of the diagnostic imaging study

- e.g. diagnostic information vs.. radiation dose

comparison of methods or systems

- computed radiography vs. plain film
- MRI vs. US

Motivation

Two steps in the radiologic process:

1. image production and display
   physical measures (MTF, NPS, NEQ, DQE)
2. image interpretation
   observer studies (ROC)

Physical Measures of Image Quality

What is a good (or valid) measure of image quality?

image of a mammogram

series of images
(rose 1)

Perceived Image Quality is Proportional to SNR

$$SNR = C\sqrt{AQ}$$

where: SNR = signal-to-noise ratio
C = image contrast of the object
A = area of the object
Q = number of quanta per unit area

Outline of Talk

Image Quality Metrics

- what are they?
- what do they mean?
- how are they determined?

Rose Model

$$SNR = C\sqrt{AQ}$$

Assumptions: (ideal detector)

- no blurring
- no added noise
- perfect absorption of incident quanta

Why Work in the Spatial Frequency Domain

performance of a detector depends on the object being imaged

a single analysis in the spatial frequency domain can be used to predict performance of all possible objects

all real objects can be decomposed into sine waves of different amplitudes, frequencies, and phases

computation in spatial frequency domain is easier than in the spatial domain

(multiplication vs. convolution)

Spatial Resolution

can be characterized by limiting resolution

measured using bar pattern

a more complete description is given by modulation transfer function (MTF)

image

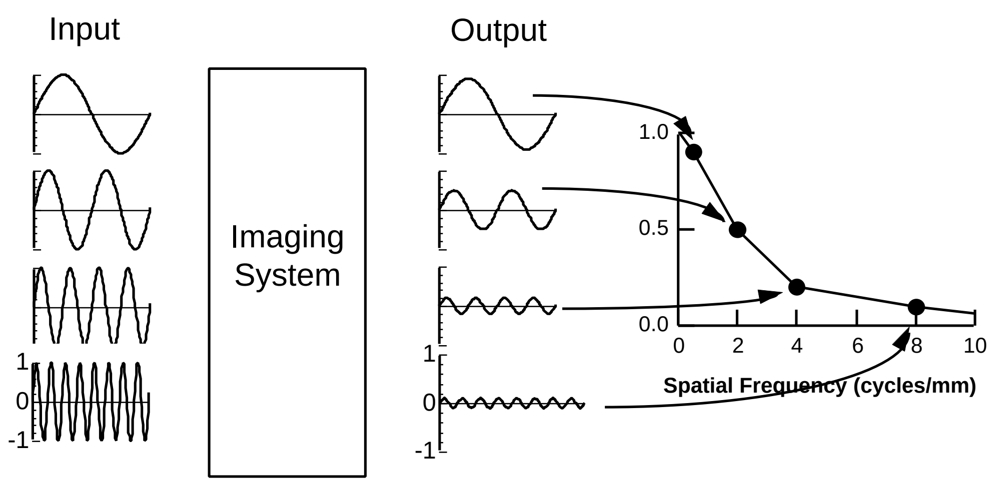
rossmann beads and needles

need MTF for intermediate freq; limiting resolution is for high freq only

Outline of Talk

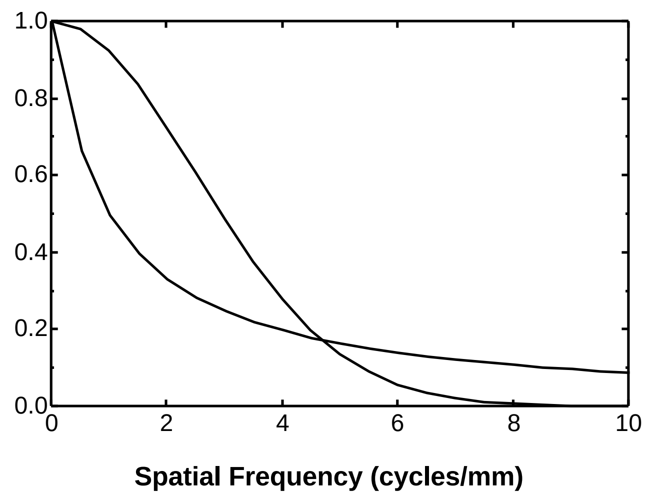
Image Quality Metrics
- what are they?
- what do they mean?
- how are they determined?

Measuring MTF (conceptually)



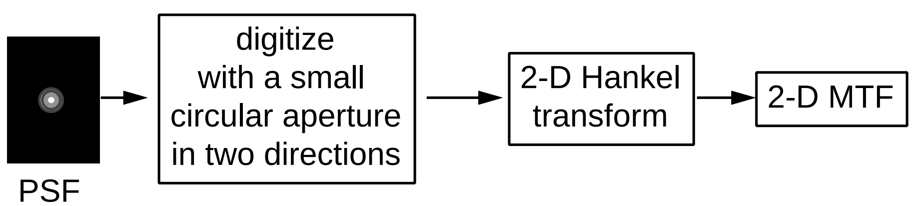
measures change in the amplitude of sine waves

MTF Curves

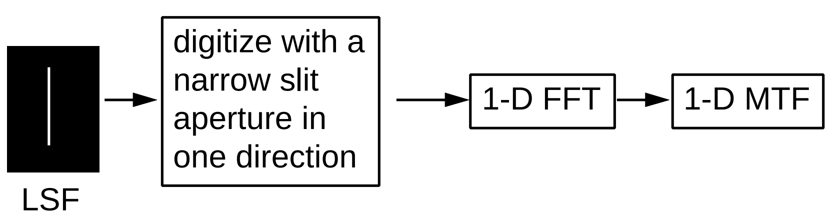


Measuring MTF (theoretically)

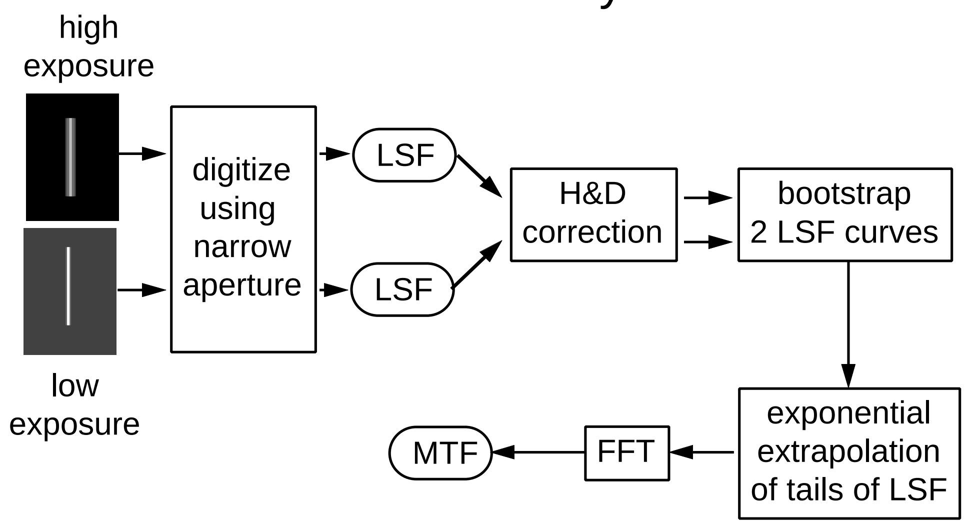
a POINT is composed of all spatial frequencies



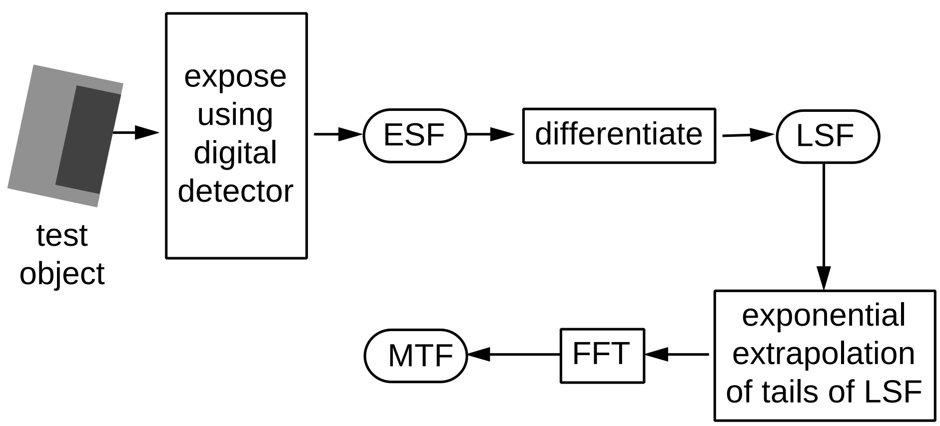
a LINE is composed of all spatial frequencies in one direction and zero frequency in the other



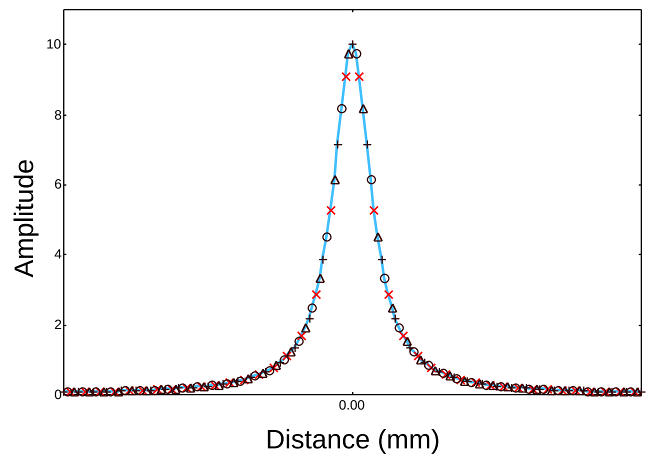
Measuring MTF (experimentally)
Screen-Film Systems



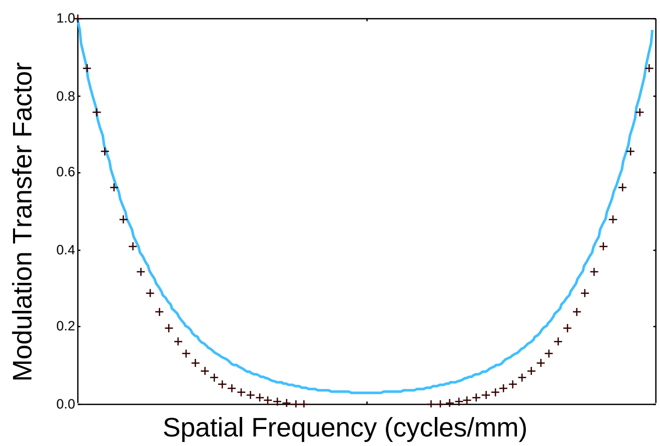
Measuring MTF (experimentally)
Digital Detectors (Pre-Sampled)



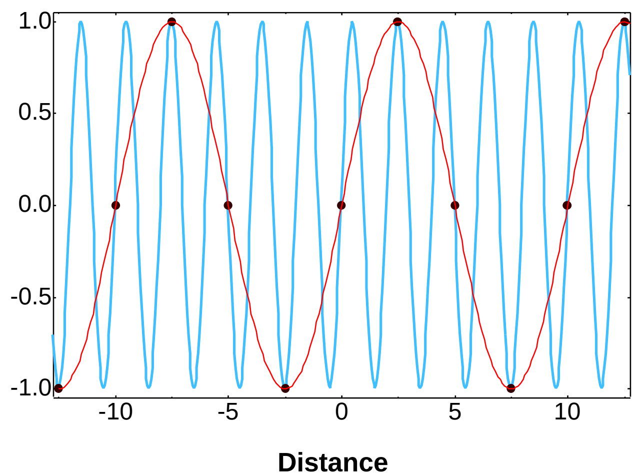
Oversampling the LSF



Aliasing



Aliasing



MTF of Digital Detectors

non-isotropic --> 2-D display is necessary

MTF in orthogonal directions can be different

Noise

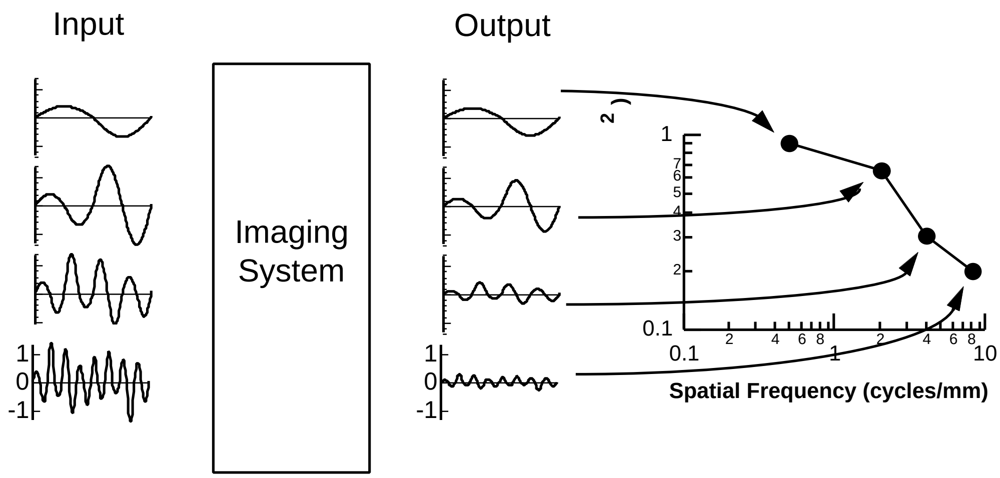
noise can be characterized by standard deviation in the output image

a more complete description is given by the noise power spectrum

noise image

same standard deviation, but different texture

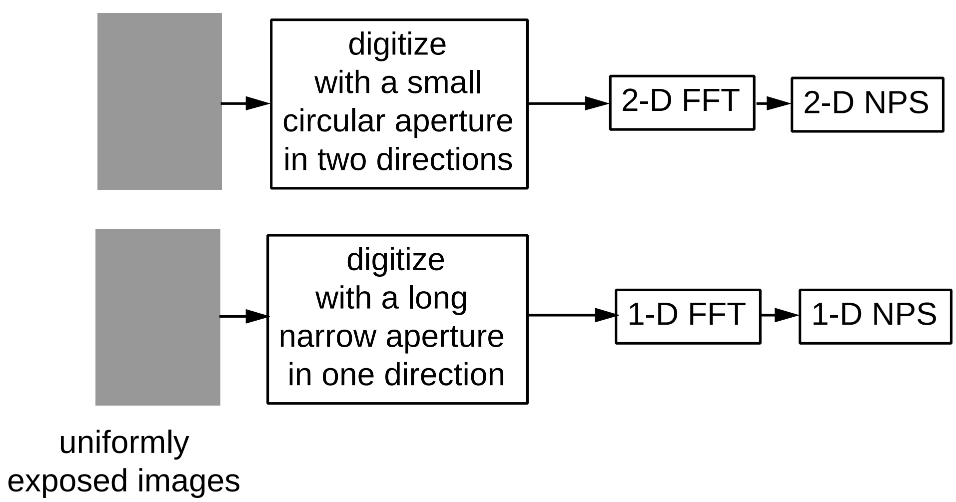
Measuring NPS (conceptually)



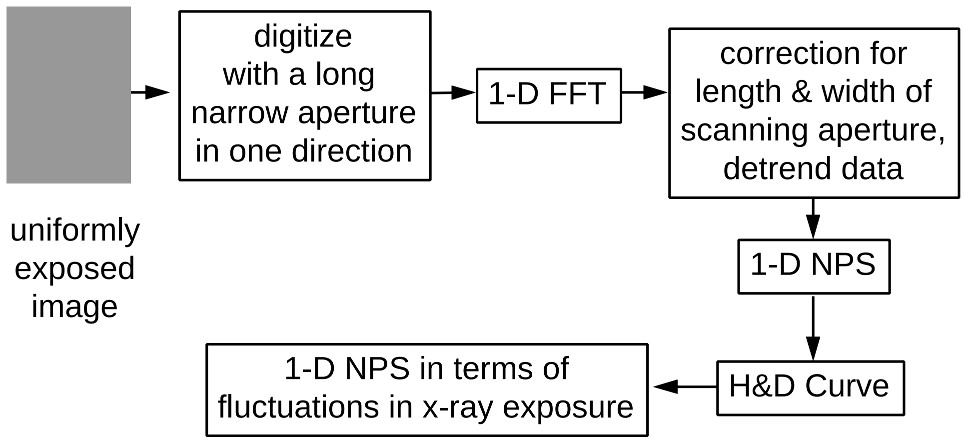
Measure change in the
variation in the amplitude of sine waves

Measuring NPS (theoretically)

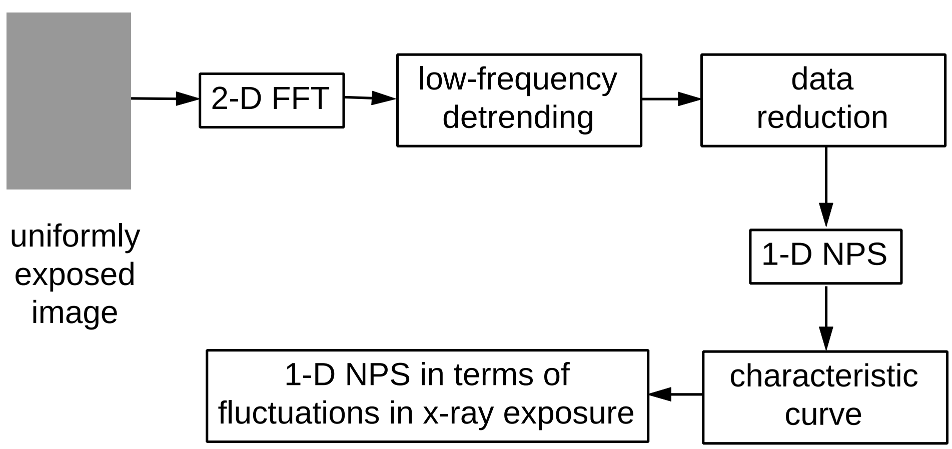
a uniform x-ray exposure contains noise at all spatial frequencies



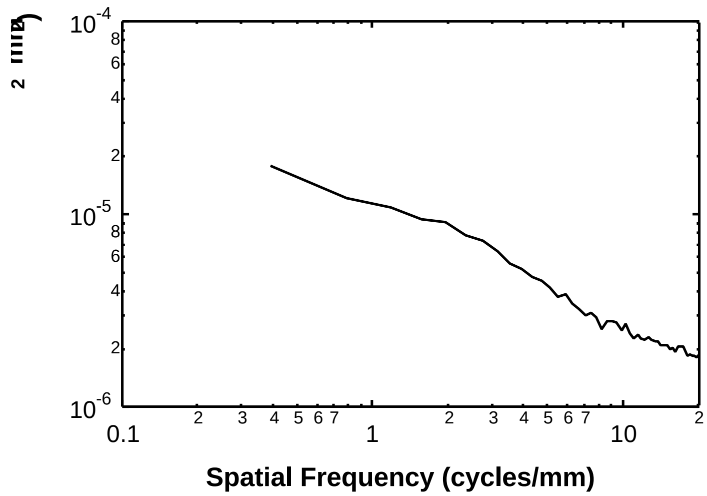
Measuring NPS (experimentally)



Measuring NPS (experimentally) Digital Detector



Typical NPS



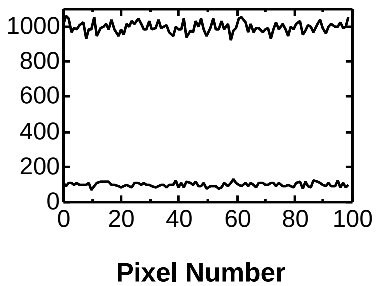
Alternate Methods for Measuring Noise Power Spectra

Fourier Transform of autocovariance function

analog method

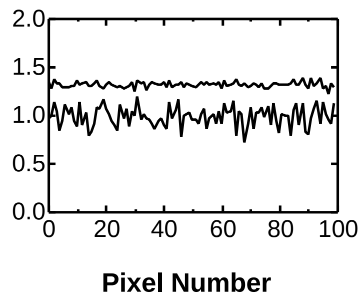
Paradox

Linear Conversion
(Digital Detector)



noise increases with exposure

Logarithmic Conversion
(Screen-Film System)



- noise decreases with exposure

Solution

Digital Detector

$I = kQ$

$dI = kdQ$

noise $\alpha \quad \overline{Q}$

Screen-Film Systems

$D = G \log(Q) + D_o$

$dD = G\, d\log(Q)$

$= G \log_{10}e\, d\ln Q$

$= G \log_{10}e\, dQ/Q$

noise $\alpha\ (Q)^{-0.5}$

assuming Poisson noise, $dQ = \sqrt{\overline{Q}}$

Signal-to-Noise Ratio

Photon Counting

signal = ΔQ

$= k\Delta Q$

SNR = $\Delta Q\ (Q)^{-0.5}$

$= C\ (Q)^{0.5}$

Screen-Film Systems

signal = ΔD

$= G\ \Delta[\log(Q)]$

$= G \log_{10}e\ \Delta Q/Q$

SNR = $\Delta Q/Q\ (Q)^{0.5}$

$= C\ (Q)^{0.5}$

where $C = \Delta Q/Q$, the radiation contrast of the object

Signal-to-Noise Ratio

can be characterized

a more complete description is given by NEQ (noise equivalent quanta) *image*

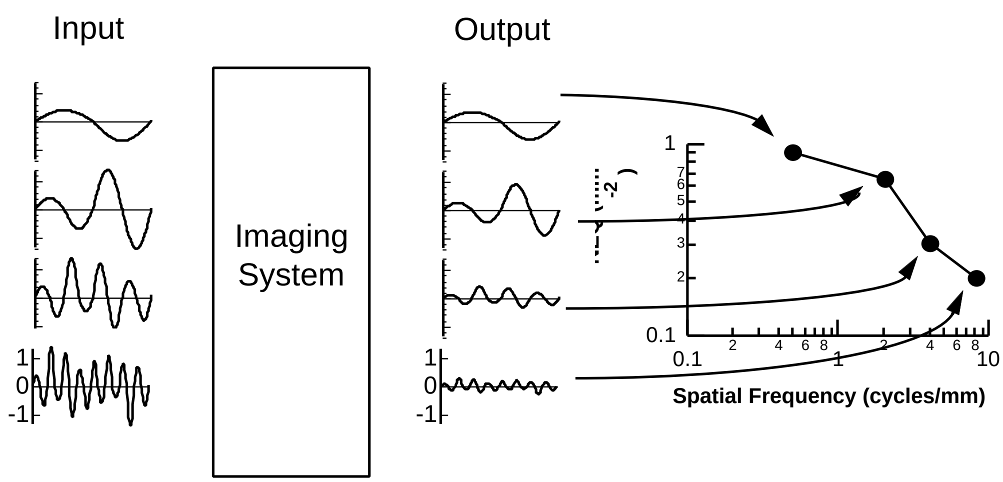
CD phantom of digital system

digital low MTF low noise

film high MTF High noise

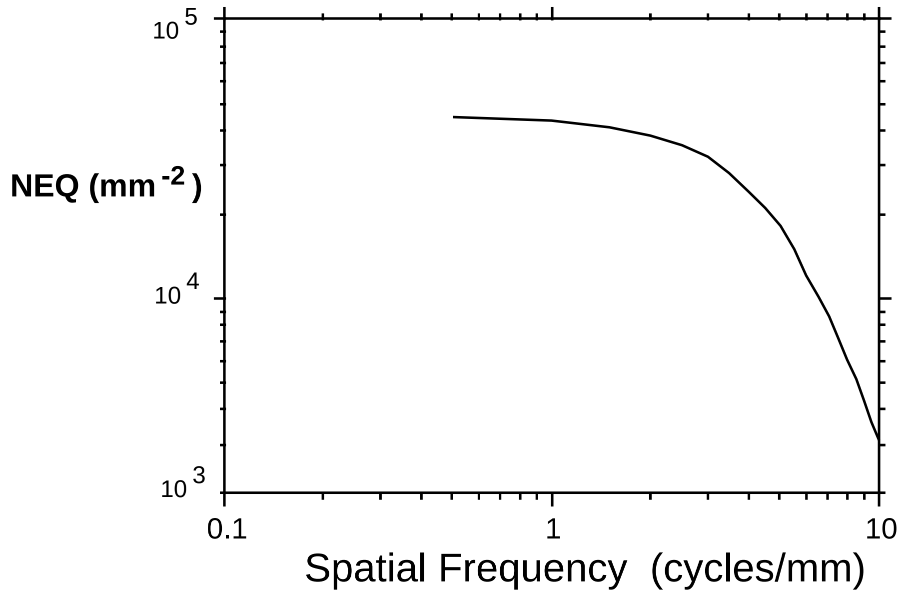
digital better

Measuring NEQ (conceptually)



Measure change in the <u>mean amplitude</u> and in the <u>variation in the amplitude</u> of sine waves

Noise Equivalent Quanta



Noise Equivalent Quanta (NEQ)

Definition:

$$NEQ(\omega) = Q\, DQE(\omega)$$

Q = # of quanta incident on the detector per unit area
(assumes unit contrast)

Detective Quantum Efficiency (DQE)

Definition:

$$\mathrm{DQE}(\omega) \quad \frac{\overline{\Delta Q^{2}(\omega)}}{\overline{\Delta O^{2}(\omega)}} \left(\frac{dO}{dQ}\right)^{2}$$

where
- ω = spatial frequency
- $\overline{\Delta O^{2}}$ = mean-squared variation in the output
- $\overline{\Delta Q^{2}}$ = mean-squared variation in the input
- $\frac{dO}{dQ}$ = gain of system

Interpretation of DQE

$$\mathrm{DQE}(\omega) = \frac{\mathrm{SNR}^{2}_{out}(\omega)}{\mathrm{SNR}^{2}_{in}(\omega)}$$

$\mathrm{SNR}_{out}(\omega)$ = SNR in the output image

$\mathrm{SNR}_{in}(\omega)$ = SNR incident on the detector

characterizes the efficiency of information transfer from the input to the output of the system

allows comparison to an ideal system

ranges from 0 to 1.0

Interpretation of NEQ

$$NEQ(\omega) = Q\ DQE(\omega)$$

For a noise-limited system, $SNR_{in}^2 = Q$

$$NEQ(\omega) = SNR_{in}^2(\omega)$$

is the number of quanta that an ideal detector would have needed to yield the same SNR

absolute measure of image quality

ranges from 0 to infinity

assumes unit contrast

How to Calculate DQE (general)

$$DQE(\omega) = \frac{Q\ MTF^2(\omega)}{W(\omega)} \left(\frac{dO}{dQ}\right)^2$$

where $MTF(\omega)$ = MTF of detector

$W(\omega)$ = noise power spectrum of image

$\frac{dO}{dQ}$ = gain of the system

How to Calculate DQE (screen-film system)

$$\gamma \quad \frac{dD}{d(\log_{10}Q)} = \frac{Q}{\log_{10}e}\frac{dD}{dQ}$$

$$\frac{dD}{dQ} = \frac{\gamma \log_{10}e}{Q}$$

$$DQE(u) = \frac{\gamma^2 (\log_{10}e)^2 MTF^2(u)}{Q W(u)}$$

u = one dimensional spatial frequency

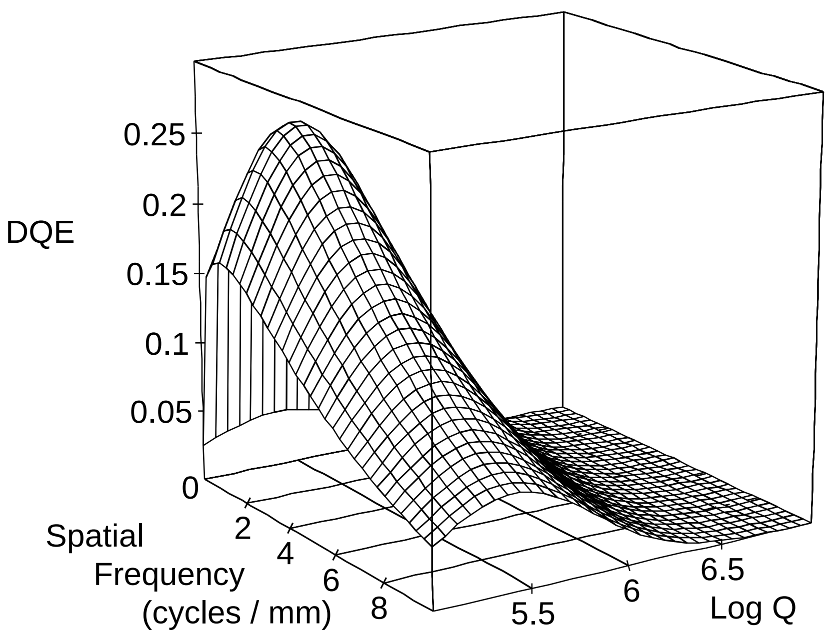
Exposure Dependence

screen-film systems are non-linear

NEQ and DQE are functions of both spatial frequency and x-ray exposure

$$NEQ(\omega,Q) = \frac{\gamma(Q)^2 (\log_{10}e)^2 MTF^2(\omega)}{W(\omega,Q)}$$

H&D curve



DQE
0.25
0.2
0.15
0.1
0.05
0
Spatial Frequency (cycles / mm)
2
4
6
8
5.5
6
6.5
Log Q

Things to Remember

DQE comparisons assume equal SNR_{in}

may not be true: x-ray exposure, kVp

$SNR_{in} = C \quad \overline{Q}$

DQE analysis assumes shift-invariant system

DQE & NEQ are measures of SNR

if image is not noise limited, but contrast limited, a system with higher NEQ may not produce a better image

information

Relationship Between SNR and NEQ

$$SNR = \left[\int |S(\vec{\omega})|^2 NEQ(\vec{\omega})\, d\vec{\omega} \right]^{1/2}$$

where $S(\vec{\omega})$ is the spatial frequency spectrum of the object

Summary

NEQ and DQE are useful parameters for characterizing and understanding medical imaging systems

NEQ and DQE can serve as a basis for comparing different imaging conditions and modalities

NEQ may be useful in furthering our understanding of image perception

Recommended Reading

(1) ICRU Report 41: Modulation transfer function of screen-film systems.

(2) BRH Report: MTF's and Wiener spectra of radiographic screen-film systems.

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(7) R. A. Sones, G. T. Barnes: A method to measure the MTF of digital x-ray systems. Med Phys **11**: 166 (1984).

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